

Understanding algebraic manipulation: Analysis of the actions of sighted and non-sighted students

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Abstract

Doing mathematics still remains a challenge for non-sighted students. Communicating with their sighted peers or teachers while doing math also presents difficulties since they use different representation supports. Interaction on accessible mathematical interfaces has had promising results in browsing using braille and audio, but support for doing math requires further development. To date, there aren't many options regarding accessible support for manipulating expressions as it is regularly practised in the learning stage.

We have carried out an experiment with sighted and non-sighted students with the aim to better understand the actions they perform while solving basic algebraic expressions, in order to design the interaction in a multimodal interface for doing math. Sighted and non-sighted participants solved four algebraic expressions orally, and sighted participants solved them in write as well. The results showed that the intentions of the participants were in conformity with the systematic application of algebraic rules, regardless of visual ability. It is suggested that success in the interaction design of the interface will depend on the degree of direct access to the expression terms, and the ease of use of features intended to minimise memory load.

1 Introduction

The study of mathematics and science in general has been a challenge for people with blindness. The scarcity of printed and digital study material contributes to the difficulty of adaptation in a mainstream environment. On the other hand, communication with sighted peers and teachers presents yet another barrier, given the differences in representation support. The use of the computer and adaptive technologies have promoted the production of material and improved the access to mathematic and scientific contents for people with blindness.

The access to mathematical contents for both browsing and solving has been the subject of several research and development projects. The Math Genie [9] [10] uses speech to convey the structure of an expression and allows navigation. Lambda uses audio and a 8-dot proprietary braille for output. The MaWEn prototypes [2] use a synchronised view for sighted and non-sighted users, allows browsing the expressions, extend and collapse its nodes. In addition, MaWEn provides braille output in different mathematical codes. These developments have enabled students with blindness the reading and writing of expressions; however, the support for doing mathematics in the learning stage has yet to achieve its full potential.

Based on the analysis of school books, [19] suggest that the essential operations involved in working with basic algebra are:

- Simplification of multiple terms, i.e. $3x + 7 + x + 2x - 10$
- Solving a linear equation, i.e. $x + 3 = 2(x + 4)$
- Polynomial multiplication and division, i.e. $(3x - 2)(x^2 - 5x + 2)$
- Manipulation of fractions
- Operations with vectors and matrices
- Resolution of a system of linear equations

From the software mentioned above, Lambda and the MaWEn prototypes include some features for performing such operations. Lambda allows copying and pasting expressions for modifying its terms, and offers a method for polynomial division [6]. Some tests have been carried out with MaWEn manipulation and simplification assistants [11][18]; however, the test results showed that their use was not very straightforward.

On the other hand, there exist software for solving algebraic equations automatically, in a similar manner as in calculators. Such software are commonly known as Computer Algebra Systems (CAS), and their objective is to facilitate symbolic computation; therefore they are not recommended for students who begin to learn the ropes of basic algebra. In fact, it is acknowledged that this type of automatic systems could represent an obstacle to the development of algebraic symbolism in students [4][12]. Examples of CAS are MuPAD, Sage, Axiom and Mathematica.

While there is accessible software available for performing automatic calculations whose only goal is to find a result, there aren't many options regarding support for manipulating the expressions as it is regularly practised in the learning stage. After having analysed the disadvantages of current CAS, the Texas Instruments company developed the Symbolic Math Guide (SMG) for the calculator TI-92. The SMG allows the user to choose the transformations that could be applied to expressions and shows the partial results, instead of finding the solution automatically; these features make it desirable to be used as a didactic support system [14]. The systems aimed for learning algebra have been specifically designed with an educational purpose, such as APLUSIX [13], PIXIE [16] and VP Algebra. Despite their didactic features, these software applications remain inaccessible for students with blindness.

2 Issues in algebraic manipulation

It is acknowledged that algebraic expressions and the transformations that can be performed on them can be easily identified and characterised [1]. In the study on algebra solving strategies conducted by [7], the actions carried on by the participants solving a linear equation could be divided in three stages:

Attraction Rearrangement of the terms with the unknown in such a way that it can be eliminated by a further operation.

Collection Elimination of the occurrences of the unknown.

Isolation Elimination of the unknown's surrounding structure.

According to the results of this study, the solution of a linear equation could involve carrying out these steps one or more times, sometimes in different order, depending on the skill of solvers.

Sighted students normally use paper and pen as an external memory while solving algebraic problems, as it was the case of participants of the study by [7]. Similarly, non-sighted users can use braille either in paper or in a braille display. In a case study, [18] have sketched the difficulties of formal manipulation faced by blind people using a braille display, namely:

1. Constant jumping between the resulting line and the preceding ones
2. Go back to specific positions in preceding lines
3. Memorise the reference terms
4. Memorise partial results that will be added to the final result
5. Browse the expression left and right of the reference term

Needless to say, the difficulties increase according the size and complexity of the expression.

However, the problem is not just a matter of providing an equivalence of representation and manipulation. Another challenge is to accommodate for communication and cooperative work, since neither of them would be able to use their usual means of representation to communicate with the other. It is suggested that a multimodal interface could suit this purpose. The characteristics of the interactions meant to perform the transformations on an interface must be carefully conceived, since such features will play an essential role in the construction of meaning [5] [8].

Some features have already been proposed as desirable to have in an interface for doing math, such as:

1. Browsing within the expression [2] [9]
2. Allowing to replace or type some elements [2]
3. Providing an overview [9] [17] [18]
4. Minimizing errors in remembering and copying [18]
5. Finding relevant spots [18]
6. Providing views for both sighted and non-sighted students [3] [11]

We have carried out an experiment to study in detail the actions of students while working with basic algebra, in order to better understand and complement the features that have been suggested, and identify the specific scenarios where they could be needed. Part of this identification includes the analysis of the memory issues that render transformations difficult to make and contribute to making errors. Following the results of the study by [7] which suggest that the involved transformations depend on the demands of the solving task, while the number and choice of operations depend on the algebraic skills of solvers, we expect similar results from non-sighted participants. We will observe if there are differences in their actions or strategies that could be related to the lack of sight. The results of our study will help us design the type of interaction and supporting features for a multimodal interface for communication and cooperation while working with basic algebra.

3 Experiment

3.1 Participants

10 sighted students and 4 non-sighted students participated in this experiment. All participants except two are Computer Science students from bachelor level; they all have already studied basic algebra. We considered it important that participants had at least the basic notions of algebra, in order to be able to carry out a strategy that would allow us to analyse their intentions.

3.2 Expressions

The expressions presented to the participants are the following:

$$2x + 3x + 4 + 5x^2 + 6 + x \quad (1)$$

$$(x + 5)(2x - 2) \quad (2)$$

$$(3a^2 + 2a + 7)(a + 5a - 4) \quad (3)$$

$$x + 2(x + 2(x + 2)) = x + 2 \quad (4)$$

Expression 4 was taken from the exercises used in [7] to analyse the strategy of resolution of equations with sighted students. This expression in particular was chosen because it allows for different ways to develop a solution, and because it was reported to be complex for the participants due to the nesting of its factors.

The expressions included one or more of the following tasks:

1. Simplify multiple terms
2. Multiply monomials, binomials and polynomials
3. Solve a linear equation

3.3 Modalities

The experiment was carried out in two modalities:

- Oral : All participants performed the operations in a non-visual manner, with the aid of the observer.
- Written : Sighted participants performed the operations on paper.

In the first case, the observer read the expression to the participant in a similar way as a speech synthesiser would do. In turn, the participant indicated the actions or operations he wanted to perform, and requested whatever information he needed in order to finish the exercise. In this modality, participants were not allowed to take notes and therefore they had to rely on their memory and on the help from the observer. It is a common belief that people who are blind have better memory than sighted people, and therefore it could be thought that they would be in a more advantageous position carrying out this exercise. In fact, the results of a recent study by [15] suggest that people who are congenitally blind develop a better serial memory than sighted people in order to compensate for their lack of sight. However, we cannot relate this result to our study for two main reasons: 1) we cannot assume that their results apply to people with non-congenital blindness, because their situations may vary, which is the case of our participants, and 2) The nature of the experiment in [15] is of memorisation throughout repetition of recall and recognition exercises, while the nature of our task is problem solving.

Participants were encouraged to share any comments on the exercise. In both modalities, the exercises were carried on individually; dialogs were recorded and then transcribed for its analysis.

Table 1: Results for expression (1)

$2x + 3x + 4 + 5x^2 + 6 + x$	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	N1	N2	N3	N4
Find common terms of a certain exponent	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Organise terms according to the exponent	*	*				*					*			
Find coefficients of a certain exponent	*													
Combine terms of a certain exponent	*	*		*							*			
Get non-processed terms				*									*	*
Copy unchanged terms to the next line	*													
Finished without errors. Oral modality	*	*	e	*	*	*	*	*	*	e	*	*	*	*
Finished without errors. Written modality	*	*	e	*	e	*	*	*	*	*	-	-	-	-

S1...S10 : Sighted students; N1...4: Non-sighted students ; e : Error

Table 2: Results for expression (2)

$(x + 5)(2x - 2)$	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	N1	N2	N3	N4
Distribution														
Get factor		*	*		*	*	*	*		*		*		
Get one of the terms from a factor	*	*			*				*	*	*	*	*	
Get pair of terms (one from each factor)													*	
Multiply directly (without indicating the products)	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Collection														
Find common terms of a certain exponent	*	*				*	*			*	*			
Combine terms of a certain exponent											*		*	
Get non-processed terms		*				*								
Copy unchanged terms to the next line	*	*												
Finished without errors. Oral modality	*	*	*	*	e	*	*	*	*	e	*	*	*	x
Finished without errors. Written modality	*	*	*	*	e	*	*	*	e	*	-	-	-	-

S1...S10 : Sighted students; N1...4 : Non-sighted students; e : Error ; x : Abandoned exercise

4 Results and discussion

In the oral modality, we have scanned through the transcribed dialogs searching for the specific actions that participants requested to perform. We searched in particular for possible differences between the intentions of sighted and non-sighted participants, while also comparing the memory issues and the errors they may have produced in the oral modality. Lastly we observed if the participants carried out the exercise successfully, and in the opposite case, we analysed the type of errors found in both modalities.

We present the results for each expression, according to the solving task that it demanded.

4.1 Expression (1). $2x + 3x + 4 + 5x^2 + 6 + x$

This expression involved simplification of multiple terms with different exponents. It is equivalent to the Collection operation [7] mentioned above. The actions that participants have demanded from the observer are shown in Table 1.

On listening the expression for the first time, all participants identified that there were terms with common factors. Some of them needed to listen the expression a second time. In order to simplify, all participants expressed the intention to find the common terms of a certain exponent. In terms of interaction, the direct request for finding common terms was more popular than the request to read the expression again to identify such terms by themselves. However, this preference could be due to the indirectness of the exercises, since they were not able to directly manipulate the expression.

Table 1 shows that 2 participants, both of them sighted, made an error in the oral modality. One was an error of sign, and the other was an omission of a term in the result. In the written modality two sighted participants omitted a term in the result.

4.2 Expression (2). $(x + 5)(2x - 2)$

This expression involved the multiplication of two binomials, one of which has a negative term, followed by simplification of multiple terms in different exponents. Operationally, the tasks involved are distribution and collection. The actions of participants are shown in Table 2.

Table 3: Results for expression (3)

$(3a^2 + 2a + 7)(a + 5a - 4)$	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	N1	N2	N3	N4
Distribution														
Get factor	*	*	*	*			*	*		*				
Get one of the terms from a factor	*	*			*	*	*	*	*		*	*		
Get pair of terms (one from each factor)	*	*						*				*		
Multiply directly (without indicating the products)			*	*	*	*	*	*	*		*	*		
Multiply abstractly, assigning letters to terms													*	
Indicate distribution $term*(factor)$	*	*												
Indicate products of every distribution										*				
Get indicated products										*				
Collection														
Find common terms of a certain exponent	*	*		*		*	*	*	*	*	*	*		
Organise terms according to the exponent											*	*		
Combine terms of a certain exponent											*		*	
Get number of terms in a factor			*		*				*					
Get the highest exponent	*										*			
Find coefficients of a certain exponent	*								*					
Copy unchanged terms to the next line			*			*		*	*					
Simplified the second factor (oral modality)						*					*			
Simplified the second factor (written modality)						*					-	-	-	
Finished without errors. Oral modality	*	*	*	e	*	e	*	e	*	x	*	e	*	x
Finished without errors. Written modality	*	*	*	e	*	*	e	*	*	e	-	-	-	-

S1...S10 : Sighted students; N1...N4 : Non-sighted students; e : Error ; x : Abandoned exercise

In order to distribute, students have requested to hear the reference terms several times, either by term, by factor, or by pairs. All participants have preferred to multiply directly, without indicating the products before. Regarding memory load, we did not observe differences between sighted and non-sighted participants; they all had trouble remembering the terms. After the distribution operation, most participants wanted to know the partial result, in order to proceed with simplification.

Participants requested the same actions for simplifying as in expression 1, though there was a slight difference in preference for finding the common terms. This time more participants were able to identify and combine terms with common exponents on listening the partial result, probably due to the reduced number of resulting terms. We suggest that as the number of terms increases, the need for requesting to find common terms will be more evident.

It was interesting to notice that some participants demand some kind of control in the collection phase. Two participants wanted to get the terms that had not been processed yet, and two requested to automatically copy or pass to the partial result the terms they had not modified. The current support of Lambda for solving is the edition option copy-paste, so that users work on the next line by changing only what they need.

Errors remain low in this exercise. Two participants made errors in the oral modality: one was arithmetic, and the other an error of control on multiplying 2 terms; the participant did not ask for the reference terms of that specific product, and multiplied $2 * 2x$ instead of $5 * 2x$. In the written modality, one error was arithmetic and the other a mistake in writing an exponent.

One of the non-sighted participants abandoned the exercise, arguing that he did not remember what to do.

4.3 Expression (3). $(3a^2 + 2a + 7)(a + 5a - 4)$

This expression is similar to the former one, only it has more terms in its factors. The terms of the second factor could be simplified at the beginning; after distribution, simplification involves the elimination of quadratic terms. As in the former expression, the tasks involved are distribution and collection. The actions that participants demanded from the observer are shown in Table 3.

Participants continue to prefer multiplying directly in the oral modality, although this time two of them decided to indicate the distributions $term*(factor)$, probably because this expression is longer and actions require greater degree of control. In the written modality no one indicated the distributions. Once more, all participants needed to ask for the terms of reference several times. Participant N2 commented: “well it’s the same principle [than equation 2], but this is a really long sort of equation (...). It’s sort of

Table 4: Results for expression (4)

$x + 2(x + 2(x + 2)) = x + 2$	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	N1	N2	N3	N4
Distribution														
Get factor					*	*				*	*			*
Get one of the terms from a factor	*					*				*				*
Get pair of terms (one from each factor)	*													
Get number of parenthesis in the expression	*				*								*	*
Get term multiplying a factor	*													
Isolate subexpression to distribute	*						*							
Multiply directly (without indicating distribution)	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Attraction														
Pass term(s) to the other member and change its sign		*	*	*		*	*		*	*	*	*	*	*
Add/subtract term(s) from both members	*										*	*	*	*
Collection														
Find common terms of a certain exponent	*	*		*		*	*	*	*	*				*
Combine terms of a certain exponent											*			
Get non-processed terms													*	
Copy unchanged terms to the next line				*	*									
Isolation														
Divide both sides by a number	*										*	*	*	*
Indicated the full expression in every line	*		*						*	*	*	*	*	*
Remembered to process the right member	*	*	*	*		*	*		*	*	*	*	*	*
Finished without errors. Oral modality	e	e	e	e	e	e	e	e	e	e	*	*	*	e
Finished without errors. Written modality	*	e	e	e	e	e	e	e	e	e	-	-	-	-

S1...S10 : Sighted students; N1...4 : Non-sighted students; e : Error

difficult to make sure that you don't miss any term... and it can be a bit challenging to keep track".

Only two participants, one sighted and one non-sighted, noticed that it was possible to simplify the second factor in the oral modality. Interestingly, from the sighted students, only the one who had simplified it in the oral modality was able to simplify it in the written one. This fact may suggest that sight, or the lack of it, is not the main factor contributing to understanding an expression and the operations related to it, and therefore to the development of an effective solving strategy.

The incidence of errors increase in the exercise, most of them happening during the distribution operation. Four participants made errors in the oral modality; the sighted participants made an arithmetic error, an error of sign, and errors due to not remembering the right terms to multiply; the non-sighted participant forgot to process the linear terms during simplification, and ended up with a final result missing the linear component. In the written modality, participants made 2 arithmetic errors and one mistake in writing an exponent.

We noticed that participants had difficulty mostly keeping track of the terms of reference and therefore of the products of the distribution. This control error was common in the oral modality, but hardly happened in writing.

Two participants, one sighted and one non-sighted, abandoned the exercise.

4.4 Expression (4). $x + 2(x + 2(x + 2)) = x + 2$

The expression we considered as the most complex involved the multiplication of nested factors, the simplification of terms and finding the value for x . The operations involved are distribution, attraction, collection and isolation. The actions that participants demanded from the observer are shown in Table 4.

In solving this expression, 12 participants chose to clear the parentheses before working with the right member; only 2 of them, both non-sighted, rearranged the terms of the right member on the left and then proceeded with simplification or distribution. This result is consistent with the study by [7], where it was found that solvers prefer to complete the clearing of parentheses before proceeding to combine terms. It was interesting to notice that by following this strategy in a non-visual fashion, 2 participants forgot that there was a right member, and left the equation unfinished. From those who did remember it, only 3 of them, sighted, indicated explicitly the full expression in every line. The same thing happened in the written modality; while working with the left member, they did not include the right member in the partial results, but only indicated it when it was time to work with it. In the phase of attraction, in which they were expected to rearrange terms between the left and right members, participants specified one

of two actions: either pass terms to the other member and change its sign, or add/subtract a term from both members. The collection operation was performed as usual. The only 4 participants who reached the phase of isolation, indicated to divide both sides by a number. The rest of them could not reach that phase because they made a distribution error which result was no longer linear.

On listening the expression for the first time, some participants were confused as to the number of parentheses involved, and others wanted to make sure they had grasped the structure of the nested parentheses: “ ...so we have three $x+2$ in brackets”, “ ...so now we have just one bracket any more, right?”. This issue may concern the need for an overview, already suggested in [9] [17] [18], but its need was not evident in simpler expressions.

It was interesting to note that this expression, the most complex in structure, was not difficult to grasp for non-sighted participants. In fact, its solution turned out to be less overwhelming than the former one, probably because it demanded less effort to keep track of the reference terms.

Errors were numerous in this case. All of the sighted participants and one of the non-sighted made some kind of error. The most common error was in working with $x + 2(x + 2)$, since most participants identified it as the multiplication of two binomials $(x + 2)(x + 2)$ and therefore ended up with a quadratic result. We expected the sighted participants to notice their distribution error during the written exercise, but even while writing $x + 2(x + 2)$, they still multiplied it as two binomials.

5 Conclusions

Interaction on mathematical interfaces for the blind has had promising results in browsing using braille and audio, but support for solving has been reported to require further development. We have conducted an experiment that helped us understand the intentions and strategies of sighted and non-sighted students while doing basic algebra. We found out that doing exercises on basic algebra demands the systematic execution of actions that depend on the solving task, and not on visual ability. In terms of memory, we did not find significant differences between the demands of sighted and non-sighted participants, since they both had trouble remembering terms and keeping track of products in long distributions, which often led to execution errors.

Following an analysis of the type of errors that participants have made in the oral modality, the most common were omission of terms and lack of control in the products of a distribution. While the omission of terms also happened in the written modality, errors due to the lack of control did not happen. Arithmetic errors were committed in both modalities. Both sighted and non-sighted students have difficulty doing mathematics without an adequate external support.

Regarding operational support for doing math, some of our results are in conformity with other works [2] [11] [18].

6 Further work

We intend to develop an interface that will facilitate expression solving. Such interface must take into account the operational and memory issues found relevant in our study. We need to allow direct access to terms, provide support for minimising memory load, and ensure visual and non-visual feedback in order to facilitate communication. These desired features must be translated into specific options in the interface.

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